ELSEVIER

Journal of Geometry and Physics 15 (1995) 281-284

JOURNAL OF GEOMETRY AND PHYSICS

Brief Communication

Decoupling of long range interactions and temporally first cause: a toy model

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Received 29 June 1993; revised 16 February 1994

Abstract

Although the question of the unification of the gravitational and electromagnetic interactions has been obscured by the unification of the electromagnetic and nuclear interactions, $SU(2)$ gravitational gauge degrees have been recently unified to the $U(1)$ electromagnetic degrees. If the resulting tracks of charges which mediate the unifying Yang-Mills field are assumed to induce a (dilation) scale invariance on the space-time geometry, the decoupling of these long range interactions, which takes place via a $U(1)$ symmetry (periodic time) breaking, could be related to the onset of an initial singularity and origin of (linear) time.

Keywords: Long range interactions; Yang-Mills field;
1991 MSC: 81 T 13, 81 V 10, 81 V 17

1. Introduction

In a recent work [1] we presented an interaction which could govern the large scale structure of the universe. This interaction is the result of an $SU(2) \times U(1)$ unification of gravitational and electromagnetic degrees of freedom, and is mediated by a field of the Yang-Mills type. This Yang-Mills field carries a charge which gathers both gravitational and Maxwellian aspects and which could be viewed as the mediator of the long range interaction.

If space-time tracks of such charges are assumed to be conformal isometries ξ (expansion field) we will show that, in the asymptotic regime, singularities of the unifying Yang-Mills field can be avoided provided the ξ orbits are closed. This defines a periodic time. Under such an assumption, gravitational and electromagnetic coupling constants can be identified, providing support to the hypothesis.

SSDI 0393-0440(94)00015-V

A decoupling of these coupling constants is necessary for the decoupling of the related interactions and implies that the ξ orbits should not be closed, thus inducing a linear time. In that case, a singularity appears in the infinite past. We ask whether such a result could shed more light on the problem of the initial singularity.

2. Mediating charges for the (unified) long range interaction

We briefly summarize the results presented in Ref. [1].

New canonically conjugate variables for gravity (the so called Ashtekar variables) have recently been introduced, which considerably simplify the formulation of General Relativity. They are defined and related to the traditional ones (q_{ij}, p_{ij}) (the metric and extrinsic curvature of a Cauchy slice Σ) via

$$q_{ij} = \sigma_i{}^{\alpha\beta}\sigma_j{}^{\mu\nu}\varepsilon_{\alpha\mu}\varepsilon_{\beta\nu}\,,$$
$$p^{ij} = -M_k{}^{\alpha\beta}\sigma^{(i}{}_{\alpha|\beta|}q^{j)k}\,,$$

where Greek indices are $SU(2)$ indices soldered to Σ.

The key step is the introduction of two connections $\pm D$ on each slice Σ:

$$\pm D_i\alpha_\mu \equiv \partial_i\alpha_\mu + \pm A^1_{i\mu}{}^\nu\alpha_\nu,$$

where connection one forms $\pm A_i{}^{1,s}$ are suitably defined [1]. If $\pm F_{ij}{}^{1,s}$ denote the corresponding curvature two-forms, Einstein's constraints become polynomial:

$$\sigma_\mu{}^{i\nu}(\pm F^1_{ij\nu}{}^\mu) = 0\,,$$
$$\sigma^i{}_\mu{}^\nu\sigma^j{}_\nu{}^\tau(\pm F^1_{ij}{}^\mu{}_\tau) = 0\,.$$

Thus, the $SU(2)$ gravitational degrees of freedom coded in the gauge connections $A^1{}_{i\mu}{}^\nu$, enable one to reformulate General Relativity as a Yang-Mills theory. Gauge connections define horizontal sections on an $SU(2)$ bundle (P_1, Π_1) over the space-time manifold (M, g).

We denote by (P_2, Π_2) the $U(1)$ Maxwellian bundle over (M, g); $A_i{}^2$ and $F_{ij}{}^2$ denote the electromagnetic potential and electromagnetic field strength on (M, g), respectively. Their pull-backs to (P_2, Π_2) define the connection and curvature 2-form of this bundle.

Bundles (P_1, Π_1) and (P_2, Π_2) can now be spliced. We define $P_1 \circ P_2 = \{(p_1, p_2) \in P_1 \times P_2 \mid \Pi_1(p_1) = \Pi_2(p_2)\}$. Let $\Pi_{12} : P_1 \circ P_2 \to M$ be given by:

$$\Pi_{12}(p_1, p_2) \equiv \Pi_1(p_1) = \Pi_2(p_2)\,.$$

For $(g_1, g_2) \in SU(2) \times U(1)$ and $(p_1, p_2) \in P_1 \circ P_2$, define $(p_1, p_2)(g_1, g_2) = (p_1 g_1, p_2 g_2)$. Then $\Pi_{12} : P_1 \circ P_2 \to M$ is a principal fiber bundle with group $SU(2) \times U(1)$. The two bundles have been spliced.

There are also projections

$$\Pi_i : P_1 \circ P_2 \to P_i\,, \quad i = 1, 2\,,$$

given by $\Pi_i(p_1, p_2) = p_i, i = 1, 2$. Certainly $\Pi^1 : P_1 \circ P_2 \to P_1$ is a principal fiber bundle with group $U(1)$ and $\Pi^2 : P_1 \circ P_2 \to P_2$ is a principal fiber bundle with group $SU(2)$. We have the following:

Theorem 1. *Let A^i be a connection for $\Pi_i : P_i \to M, i = 1,2$; then $(\Pi^1)^* A^1$ is a connection for $\Pi^2 : P_1 \circ P_2 \to P_2$, and $(\Pi^2)^* A^2$ is a connection for $\Pi^1 : P_1 \circ P_2 \to P_1$. Finally $(\Pi^1)^*\ A^1 \oplus (\Pi^2)^*\ A^2$ is a connection for $\Pi_{12} : P_1 \circ P_2 \to M$, which we shall further denote by A^{12}.*

Let $h_{ab} = g_{ab} + k_{\alpha\beta} A^{12\alpha}{}_a A^{12\beta}{}_b$ define, via k (a gauge invariant metric on the fiber), a metric on $P_1 \circ P_2$.

Theorem 2. *The Ricci tensor of $(P_1 \circ P_2, h)$ is given by:*

$$\begin{aligned} R_{jm}(h) &= R_{jm}(g) - \tfrac{1}{2} F^{12}{}_{\alpha}{}^{k}{}_{j} F^{12\alpha}{}_{km}, \\ R_{j,n+\alpha}(h) &= -\tfrac{1}{2} F^{12}{}_{\alpha}{}^{i}{}_{j,i}, \\ R_{n+\gamma,n+\nu}(h) &= -\tfrac{1}{4} C^{\alpha}{}_{\beta\gamma} C^{\beta}{}_{\alpha\nu} + \tfrac{1}{4} F^{12}{}_{\gamma k}{}^{i} F^{12k}_{\nu\ i}, \end{aligned}$$

where $C^{\gamma}{}_{\beta\alpha}$ denotes the structure constants of the Lie-algebra of the $U(1) \times SU(2)$ fiber.

Theorem 3. *Let $A = \int_{P_1 \circ P_2} R(h) dv$ denote the action integral on $P_1 \circ P_2$. Then A is extremal w.r.t. the variations on g, and on A^{12} iff:*
(i) $R_{ij}(g) - \frac{1}{2} R(g) g_{ij} = \frac{1}{2}(F^{12\alpha}{}_{hi} F^{12}{}_{\alpha}{}^{h}{}_{j} + \mathcal{A}\ g_{ij})$, the Einstein equation, is satisfied.
(ii) $\nabla_{[a}{}^{*}F^{12\alpha}{}_{bc]} = 0$, the Yang–Mills equation, is satisfied.
(The scalar curvature of $(P_1 \circ P_2, h)$ is given by:

$$R(h) = R(g) - \tfrac{1}{4} C^{\alpha}{}_{\beta\gamma} C^{\beta\gamma}_{\alpha} - \tfrac{1}{4} F^{12}{}_{\alpha}{}^{kj} F^{12\alpha}{}_{kj} \quad \textit{and} \quad \mathcal{A} = -\tfrac{1}{4} F^{12}{}_{\alpha}{}^{kj} F^{12\alpha}{}_{kj}.)$$

Theorem 4. *Along a geodesic $X(s)$ of $(P_1 \circ P_2, h)$, $A^{12}{}_a{}^{\alpha}\ dX^a/ds$ is an element of the $SU(2) \times U(1)$ Lie-algebra S, which does not depend on the parameter s, thus defining a charge attached to the geodesic path of a particle.*

3. Orbits of the mediating charges, and temporally first cause

In this section we focus on the tracks of the above $SU(2) \times U(1)$ charges. These are geodesics of $(P_1 \circ P_2, h)$. Their projection on (M, g) is the path of a particle accelerated by the field strength:

$$F^{12\alpha}{}_{hj}\, dX^j/ds.$$

We shall assume now that the tangent vector dX^j/ds is a multiple of a conformal isometry ξ on (M, g): $\mathcal{L}_\xi g = kg$.

Let $\tilde{g}_{ab} = \Omega^2 g_{ab}$ denote the conformally rescaled metric, n^a the asymptotic null generators and i^* the pull back to the null boundary of (M, g).

From $\mathcal{L}_\xi\, F^{12\alpha}{}_{ij} = 0$ and $\mathcal{L}_\xi\, C_{abc}{}^d = 0$, where C denotes the Weyl tensor of g, it is easy to deduce

$$(i^*\mathcal{E}^i) = [\,i^*\mathcal{E}^i(0)\,]\exp\left(-aku + b\int_0^u \chi(\tilde{u})\,d\tilde{u}\right),$$

and

$$(i^*\mathcal{B}^i) = [\,i^*\mathcal{B}^i(0)\,]\exp\left(-aku + b\int_0^u \chi(\tilde{u})\,d\tilde{u}\right),$$

respectively, where u denotes an affine parameter along the orbits of $i^*(\xi)$, a, b are constants $(a > 0)$, and $\mathcal{E}_i$ (resp. $\mathcal{B}_i$) denotes $F^{12\alpha}{}_{ij}n^j$ (resp. $*F^{12\alpha}{}_{ij}n^j$). Thus the "electric" and "magnetic" charges of the Yang-Mills field $F^{12\alpha}{}_{ij}$ will be finite provided the orbits of $(i^*\xi)$ are compact (periodic time).

The ratio of these charges can be identified to the analogous one available for a (source free) Maxwellian field [2], thus inducing an identification of coupling constants.

4. Conclusion

If the orbits of the unifying Yang-Mills charge induce a dilation (expansion field) on the space-time base manifold, with non compact orbits (linear time), long range interactions can decouple into gravitational and electromagnetic degrees of freedom. In that case singularities of the fields involved must be located in the infinite past, which could thus be viewed as an origin of (linear) time.

References

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